Exercise. Let ABC be an equilateral h-triangle with side 100. Show that

$$|\measuredangle_h ABC| < \frac{1}{10\,000\,000\,000}.$$

Sketch $\triangle_h ABC$ in the conformal model.

Solution: By the triangle inequality, the h-distance from B to $(AC)_h$ is at least 50. Let φ be the angle of parallelism on distance 50. Note that $|\measuredangle_h ABC| < 2\cdot\varphi$. Therefore it is sufficient to show that

$$\varphi < \tfrac{1}{2\cdot 10^{10}}. \qquad (*)$$

Applying 13.3, we get

$$\begin{aligned} 50 &= \tfrac{1}{2}\cdot\ln\tfrac{1+\cos\varphi}{1-\cos\varphi}.\\ &\Updownarrow\\ e^{100} &= \tfrac{1+\cos\varphi}{1-\cos\varphi}.\\ &\Updownarrow\\ \cos\varphi &= \tfrac{e^{100}-1}{e^{100}+1} =\\ &= 1-\tfrac{2}{e^{100}+1}.\\ &\Downarrow\\ 1-\tfrac{\varphi^2}{10} &\geqslant 1-\tfrac{2}{e^{100}+1}, && \text{(since } \cos\varphi \leqslant 1-\tfrac{\varphi^2}{10}\text{)}\\ &\Downarrow\\ \varphi &< \tfrac{\sqrt{20}}{e^{50}} <\\ &< 10^{-15}. && \text{(since } e^3 > 10\text{)} \end{aligned}$$

Whence $(*)$ follows.

(Note that $\phi = \arccos(1-\frac{2}{e^{100}+1})$. If you try calculate this value on a standard calculator, then (likely) you get zero. Evidently, the answer is close to zero, but it cannot be zero. If you try to use a more sophisticated software (for example, WolframAlpha), then you get a better estimate, say $\varphi < 4\cdot 10^{-22}$.)

Sketch: