

# Extra credit problems

Math 471

0. Find a mistake or misprint in the book. (The score depends on the type of mistake.)
1. Show that a composition of three reflections in the sides of a nondegenerate triangle does not have a fixed point.
2. Given distinct points  $A$ ,  $B$  and  $X$  make a ruler-and-compass construction of another point  $Y \in (AX)$  such that  $AY + BY = AX + BX$  and explain why it works. (You may play with the GeoGebra applet [apfzzybw](#).)
3. Let  $m$  be an indirect motion of the plane. Show that the midpoints of all line segments  $[X m(X)]$  lie on one line.
4. Let us denote by  $\rho_\ell$  the reflection across line  $\ell$ . Let  $\ell$  and  $m$  be lines on the plane and  $n$  is the reflection of  $\ell$  across  $m$ . Show that  $\rho_m \circ \rho_\ell = \rho_n \circ \rho_m$ .
5. Lines  $\ell$  and  $m$  are tangent to two circles of radiuses  $r$  and  $R$  on such a way the circles are on one side of  $\ell$  and on different sides of  $m$ . Let  $A$  and  $B$  be tangential points of  $\ell$  and  $Q$  be the point of intersection  $\ell$  and  $m$ . Show that  $QA \cdot QB = R \cdot r$ .
6. Given a line segment with marked midpoint, perform a ruler-only construction a line through a given point parallel to the the segment. Explain why it works. (You may play with the GeoGebra applet [qrmffpkv](#).)