

**Problem.** Let  $\Gamma_1$  and  $\Gamma_2$  be two distinct circles with centers at  $O_1$  and  $O_2$  and radii  $r_1$  and  $r_2$ , and let  $d = O_1O_2$ . Show that  $\Gamma_1$  is tangent to  $\Gamma_2$  if and only if

$$d = r_1 + r_2 \quad \text{or} \quad d = |r_1 - r_2|.$$

Let  $P$  be the common point of the circles  $\Gamma_1$  and  $\Gamma_2$ .

WLOG assume  $r_2 \geq r_1$ .

Since the circles are tangent  $\angle O_1PO_2 = 90^\circ$

By the Pythagorean theorem

$$d^2 = r_1^2 + r_2^2 \quad \text{and}$$

$$r_2^2 = d^2 - r_1^2 = (d+r_1)(d-r_1).$$

Therefore, either  $r_2 = d + r_1$  or  $r_2 = d - r_1$ .

Hence the statement follows.