List of questions for the midterm

1. Inverse function theorem.

2. Sard's lemma.

3. Degree modulo 2 and integer degree; their homotopy invariance.

4. Construction of partition of unity.

5. Brouwer fixed-point theorem.

6. Thom's traversality theorem, intersection number.

7. Whitney embedding theorem (for closed manifolds).

8. Vector fields as sections of tangent bundle: integral curves, flows, straightening lemma.

9. Vector fields as a differential operator: Lie bracket, Jacobi identity.

10. Straightening lemma for commuting vector fields.

11. Lie derivative of tensor fields: defintions and proof of identities

$$\begin{aligned} \mathcal{L}_X(\alpha \otimes \beta) &= (\mathcal{L}_X \alpha) \otimes \beta + \alpha \otimes (\mathcal{L}_X \beta), \\ \mathcal{L}_X \text{Contraction} &= \text{Contraction}\, \mathcal{L}_X, \\ \mathcal{L}_{X+Y} &= \mathcal{L}_X + \mathcal{L}_Y, \\ \mathcal{L}_X \mathcal{L}_Y - \mathcal{L}_Y \mathcal{L}_X &= \mathcal{L}_{[X,Y]}. \end{aligned}$$

12. Grassmann algebra and its existence (tensor interpretation).

13. Differential forms: definition, Lie, external, and internal derivative, their product rules, pullback and its relation to wedge product and external differential ($f^*(\alpha \wedge \beta) = f^*\alpha \wedge f^*\beta$ and $f^*d = df^*$).

14. Cartan's magic formula.

15. Stokes' theorem, closed and exact forms.

16. De Rham cohomology algebra: definitions, an example of calculations via symmetry.

17. Homotopy invariance of De Rham cohomology, Poincaré's lemma.

18. Mayer–Vietoris sequence: formulation + an application.

19. Top cohomology. Cohomological definition of degree.

20. Moser's theorem via Moser's trick.

21. Morse theory: degenerate and nondegenerate critical points, existence of Morse function (for closed manifolds).

22. Morse theory: product structure at noncritical levels.

23. Morse lemma.

24. Handle decomposition: rearrangement of handles (reodering critical levels according to the index), handle body decomposition of 3-manifolds (Heegaard splitting).